© INTERNATIONAL JOURNAL FOR RESEARCH PUBLICATION & SEMINAR
ISSN: 2278-6848 | Volume: 07 Issue: 04 | July – September 2016
Paper is available at www.jrps.in | Email : info@jrps.in

Study of probability theory and probability models for dice game of craps

[1]Seema Rani, lohanpoonam@gmail.com

Introduction : Probability is the language of uncertainty. Using statistics, we can better predict the outcomes of random phenomena over the long term – from the very complex, like weather, to the very simple, like a coin flip, or of more interest to gamblers, a dice toss. A probability is a numerical value assigned to a given event A. The probability of an event is written P(A), and describes the long-run relative frequency of the event. The first two basic rules of probability are the following:

> ***Rule 1:*** *Any probability P(A) is a number between 0 and 1* ($0 \leq P(A) \leq 1$).
> ***Rule 2:*** *The probability of the sample space S is equal to 1* ($P(S) = 1$).

The sample space S for a probability model is the set of all possible outcomes. An event A is a subset of the sample space S.

Disjoint. : If two events have no outcomes in common, then they are called ***disjoint.*** For example, the possible outcomes of picking a single marble are disjoint: only one color is possible on each pick. The addition of probabilities for disjoint events is the third basic rule of probability:

> ***Rule 3:*** *If two events A and B are disjoint, then the probability of either event is the sum of the probabilities of the two events:* $P(A \text{ or } B) = P(A) + P(B)$.

Union The chance of *any* (one or more) of two or more events occurring is called the union of the events. The probability of the union of disjoint events is the sum of their individual probabilities.

> ***Rule 4:*** *The probability that any event A does not occur is* $P(A^c) = 1 - P(A)$.

Independence : Consider two events which might occur in succession, such as two flips of a coin. If the outcome of the first event has no effect on the probability of the second event, then the two events are called independent. For two coin flips, the probability of getting a "head" on either flip is 1/2, regardless of the result of the other

© INTERNATIONAL JOURNAL FOR RESEARCH PUBLICATION & SEMINAR
ISSN: 2278-6848 | Volume: 07 Issue: 04 | July – September 2016
Paper is available at www.jrps.in | Email : info@jrps.in

flip. The fourth basic rule of probability is known as the multiplication rule, and applies only to independent events:

> ***Rule 5:*** *If two events A and B are independent, then the probability of both events is the product of the probabilities for each event:*
> *P(A and B) = P(A)P(B).*

The chance of *all* of two or more events occurring is called the ***intersection*** of events. For independent events, the probability of the intersection of two or more events is the product of the probabilities.

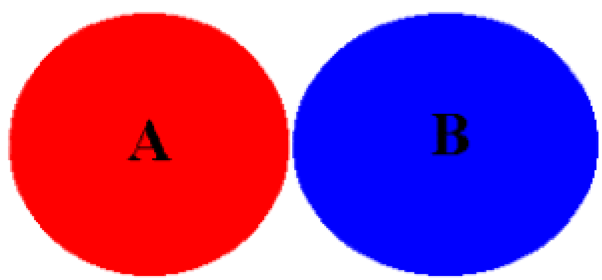
In the case of two coin flips, for example, the probability of observing two heads is 1/2*1/2 = 1/4. Similarly, the probability of observing four heads on four coin flips is 1/2*1/2*1/2*1/2 = 1/16.

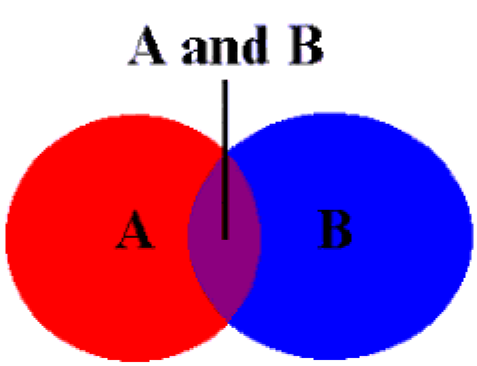
conditional probabilities : The probability of the intersection of two or events which are not independent is determined using conditional probabilities.

Venn Diagrams

A useful graphical tool for studying the complements, intersections, and unions of events within a sample space S is known as a Venn diagram. In such a diagram, events are drawn as regions that may or may not overlap.

In the Venn diagram to the right, events A and B are disjoint. Suppose, for example, event A is drawing a red marble from a bowl of five differently colored marbles, and event B is drawing a blue marble. These events cannot both occur, so there is no overlapping area.



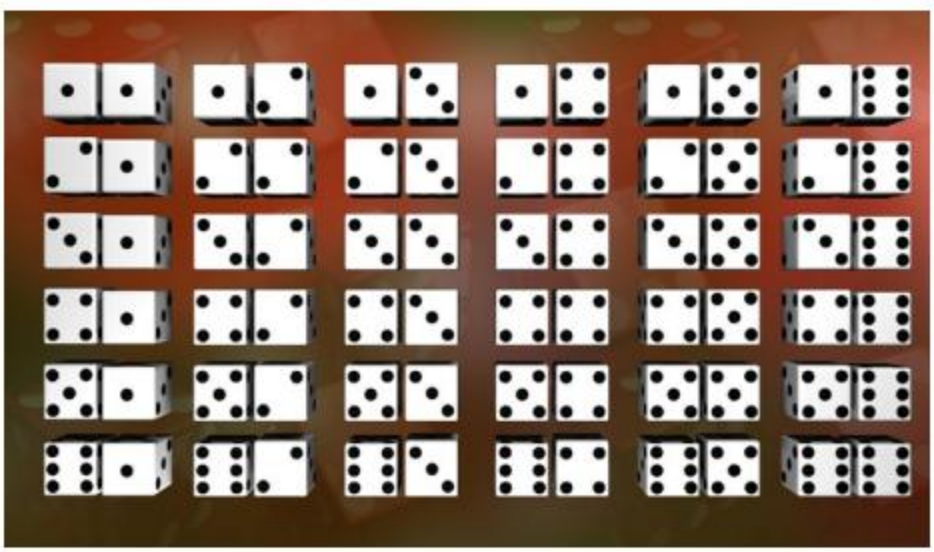


In the Venn diagram on the left, events A and B are not disjoint. This means that it is possible for both events to occur, and the overlapping area represents this

© INTERNATIONAL JOURNAL FOR RESEARCH PUBLICATION & SEMINAR
ISSN: 2278-6848 | Volume: 07 Issue: 04 | July – September 2016
Paper is available at www.jrps.in | Email : info@jrps.in

possibility. Suppose, for instance, there are 3 red marbles and two blue marbles in a bowl. Two marbles are to be drawn from the bowl, one after the other. After the first draw, the marble drawn is returned to the bowl. Define event A to be drawing a red marble from the bowl on the first draw, and define event B to be drawing a blue marble on the second draw. The occurence of event A is represented by the red area, the occurence of event B is represented by theblue area, the occurence of both events is represented by the overlapping area (also known as the intersection of the two events), and the occurence of either event is represented by the entire colored area (also known as the union of the two events).

Dice Game of Craps : the probability model for the popular dice game of craps. Depending on how players bet, money is made or lost based on the 'shooter' rolling certain numbers and not others. Assuming they are playing with fair dice, smart gamblers want to know the probability of any particular roll coming up. Here's where we start building that probability model. First, we define the sample space, S, the set of all possible outcomes. As you can see from Figure, rolling two six-sided dice means we have 36 possible outcomes.



Next, we assign probabilities to each of the possible outcomes in our sample space. Each roll is independent, meaning that the occurrence of one doesn't influence the probability of another. If the dice are perfectly balanced, all 36 outcomes are equally likely. In the long run, each of the outcomes would come up 1/36th of the rolls. So, the probability for each outcome is 1/36 or approximately 0.0278 – so, each outcome occurs roughly 3% of the time. Probabilities are always between 0 and 1, with those closer to 0 less likely to happen and those closer to 1 more likely to happen. The sum of the probabilities of all the possible outcomes in a sample space always equals 1.

For games such as craps, gamblers are more worried about the sum of the dice. So, the sample space they are interested in for rolling two dice looks like this: S = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} While each roll pictured in Figure 19.1 has an equal chance of occurring, that's not true for each sum between 2 and 12. For example, there is only one way to roll a two and only one way to roll a twelve. So, each of those outcomes has a 1/36th chance of occurring. But there are six ways to roll a seven. So, the gambler has a 6/36th or 1/6th chance of rolling a seven, which is about a 17% chance. The probability model for how many spots are going to turn up when a player rolls two dice is given in Table. A probability model is made up of all the possible outcomes together with the probabilities associated with those outcomes.

Spots	2	3	4	5	6	7	8	9	10	11	12
Probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

By tweaking the rules for what rolls pay out in what way, the casino can use the probability model to ensure that over the long term, no one will beat the house. For example, in craps the most common roll – a seven – is the one that instantly loses the round once it's underway. Suppose we wanted to know the probability of rolling anything OTHER than a seven, P(not 7). We can use the Complement Rule to figure this out. Complement Rule For any event A, P(A does not occur) = 1 – P(A). So, P(not 7) = 1 – P(7) = 1 – 1/6 = 5/6. The probability model in Table 19.1 provides plenty of other examples as well. Let's say one gambler placed separate bets on 4 and 5. He wants the next roll to add up to one or the other of those two numbers. How do we determine his chances for winning? First of all, these two events, rolling a 4 or rolling a 5, are what statisticians call mutually exclusive events, which means that these two events have no outcomes in common. Because these events are mutually exclusive, we can use the Addition Rule for Mutually Exclusive Events to figure out the gambler's chances of winning. Addition Rule If A and B are mutually exclusive events, then P(A or B) = P(A) + P(B). In this case, P(4 or 5) = P(4) + P(5) = 3/36 + 4/36 = 7/36. So, our gambler has about a 19% chance of winning. Craps players can also bet that the shooter will roll a number 'the hard way,' meaning by rolling doubles. Let's say one gambler bets the shooter will roll six the hard way. What are the chances that this bet will pay off on the next roll? We can figure this out using the Multiplication Rule, which says that if two events are independent, we can find the probability that they both happen by multiplying their individual probabilities. Multiplication Rule If A and B are independent, then P(A and B) = P(A)P(B). The roll of one die is our event A and the roll of the other is our event B. The two events are independent because whatever is rolled on one die does not affect what is rolled on the other die.

Hence, we need to calculate P(3 and 3) = P(3)P(3). Rather than the probability model we have been using for rolling two dice at once, we need a new one for the probabilities of each number coming up in the roll of a single die.

Spots	1	2	3	4	5	6
Probability	1/6	1/6	1/6	1/6	1/6	1/6

Now, we finish calculating the probability of rolling a six the hard way: P(3 and 3) = P(3)P(3) = (1/6)(1/6) = 1/36. It is now clear that no matter how skilled you get in using probability models, the house will probably win if you keep on playing.

References :

1. https://www.learner.org/courses/againstallodds/pdfs/AgainstAllOdds_StudentGuide_Unit19.pdf
2. http://www.stat.yale.edu/Courses/1997-98/101/probint.htm
3. http://www.math.uwaterloo.ca/~dlmcleis/book/appendixa1.pdf
4. http://math4allview.appspot.com/view?repo=m4a_en&parent=math4all.zoso.nl/mathlevels/view/saba-a/23&comp=ha-g1&subcomp=ha-g14
5. http://qmplus.qmul.ac.uk/course/view.php?id=3053