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Study of number theory and its historical development

[1]Seema Rani, lohanpoonam@gmail.com
[2]Amita , Assistant professor , Govt. College, Julana

Introduction : Number theory, branch of mathematics concerned with properties of the positive integers (1, 2, 3, …). Sometimes called "higher arithmetic," it is among the oldest and most natural of mathematical pursuits. Number theory has always fascinated amateurs as well as professional mathematicians. In contrast to other branches of mathematics, many of the problems and theorems of number theory can be understood by laypersons, although solutions to the problems and proofs of the theorems often require a sophisticated mathematical background

Until the mid-20th century, number theory was considered the purest branch of mathematics, with no direct applications to the real world. The advent of digital computers and digital communications revealed that number theory could provide unexpected answers to real-world problems. At the same time, improvements in computer technology enabled number theorists to make remarkable advances in factoring large numbers, determining primes, testing conjectures, and solving numerical problems once considered out of reach.

Modern number theory is a broad subject that is classified into subheadings such as elementary number theory, algebraic number theory, analytic number theory, geometric number theory, and probabilistic number theory.

Historical developments :

The ability to count dates back to prehistoric times. This is evident from archaeological artifacts, such as a 10,000-year-old bone from the Congo region of Africa with tally marks scratched upon it—signs of an unknown ancestor counting something. Very near the dawn of civilization, people had grasped the idea of "multiplicity" and thereby had taken the first steps toward a study of numbers.

- It is certain that an understanding of numbers existed in ancient Mesopotamia, Egypt, China, and India, for tablets, papyri, and temple carvings from these early cultures have survived. A Babylonian tablet known as Plimpton 322 (c. 1700 bc) is a case in point. In

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modern notation, it displays number triples x, y, and z with the property that $x^2 + y^2 = z^2$. One such triple is 2,291, 2,700, and 3,541, where $2{,}291^2 + 2{,}700^2 = 3{,}541^2$. This certainly reveals a degree of number theoretic sophistication in ancient Babylon.

- Pythagoras (c. 580–500 bc) : According to tradition, Pythagoras (c. 580–500 bc) worked in southern Italy amid devoted followers. His philosophy enshrined number as the unifying concept necessary for understanding everything from planetary motion to musical harmony. Given this viewpoint, it is not surprising that the Pythagoreans attributed quasi-rational properties to certain numbers.
  For instance, they attached significance to perfect numbers—i.e., those that equal the sum of their proper divisors. Examples are 6 (whose proper divisors 1, 2, and 3 sum to 6) and 28 (1 + 2 + 4 + 7 + 14).
- The Greek philosopher Nicomachus of Gerasa(flourished c. ad 100), writing centuries after Pythagoras but clearly in his philosophical debt, stated that perfect numbers represented “virtues, wealth, moderation, propriety, and beauty.” (Some modern writers label such nonsense numerical theology.)
- In a similar vein, the Greeks called a pair of integers amicable (“friendly”) if each was the sum of the proper divisors of the other. They knew only a single amicable pair: 220 and 284. One can easily check that the sum of the proper divisors of 284 is 1 + 2 + 4 + 71 + 142 = 220 and the sum of the proper divisors of 220 is 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284.

- Euclid gave a version of what is known as the unique factorization theoremor the fundamental theorem of arithmetic. This says that any whole number can be factored into the product of primes in one and only one way. For example, 1,960 = 2 × 2 × 2 × 5 × 7 × 7 is a decomposition into prime factors, and no other such decomposition exists. Euclid’s discussion of unique factorization is not satisfactory by modern standards

- Diophantus : Of later Greek mathematicians, especially noteworthy is Diophantus of Alexandria(flourished c. 250), author of Arithmetica. This book features a host of

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problems, the most significant of which have come to be called Diophantine equations. These are equations whose solutions must be whole numbers. For example, Diophantus asked for two numbers, one a square and the other a cube, such that the sum of their squares is itself a square. In modern symbols, he sought integers x, y, and z such that $(x^2)^2 + (y^3)^2 = z^2$.

Modern number theory :

As mathematics filtered from the Islamic world to Renaissance Europe, number theory received little serious attention. The period from 1400 to 1650 saw important advances in geometry, algebra, and probability, not to mention the discovery of bothlogarithms and analytic geometry. But number theory was regarded as a minor subject, largely of recreational interest.

Pierre de Fermat

Credit for changing this perception goes to Pierre de Fermat (1601–65), a French magistrate with time on his hands and a passion for numbers. Although he published little, Fermat posed the questions and identified the issues that have shaped number theory ever since. Here are a few examples:

1. In 1640 he stated what is known as Fermat's little theorem—namely, that if p is prime and a is any whole number, then p divides evenly into ap − a. Thus, ifp = 7 and a = 12, the far-from-obvious conclusion is that 7 is a divisor of $12^7 - 12 = 35{,}831{,}796$. This theorem is one of the great tools of modern number theory.
2. Fermat investigated the two types of odd primes: those that are one more than a multiple of 4 and those that are one less. These are designated as the 4k + 1 primes and the 4k − 1 primes, respectively. Among the former are 5 = 4 × 1 + 1 and 97 = 4 × 24 + 1; among the latter are 3 = 4 × 1 − 1 and 79 = 4 × 20 − 1. Fermat asserted that any prime of the form 4k + 1 can be written as the sum of two squares in one and only one way, whereas a prime of the form 4k − 1 cannot be written as the sum of two squares in any manner whatever. Thus, $5 = 2^2 + 1^2$ and $97 = 9^2 + 4^2$, and these have no alternative decompositions into sums of squares. On the other hand, 3 and 79 cannot be so decomposed. This dichotomy among primes ranks as one of the landmarks of number theory.

3. In 1638 Fermat asserted that every whole number can be expressed as the sum of four or fewer squares. He claimed to have a proof but did not share it.
4. Fermat stated that there cannot be a right triangle with sides of integer length whose area is a perfect square. This amounts to saying that there do not exist integers x, y, z, and w such that $x^2 + y^2 = z^2$ (the Pythagorean relationship) and that $w^2 = \frac{1}{2}$(base) (height) = $xy/2$.

Number theory in the 19th century : Of immense significance was the 1801 publication of Disquisitiones Arithmeticae byCarl Friedrich Gauss (1777–1855). This became, in a sense, the holy writ of number theory. In it Gauss organized and summarized much of the work of his predecessors before moving boldly to the frontier of research. Observing that the problem of resolving composite numbers into prime factors is "one of the most important and useful in arithmetic," Gauss provided the first modern proof of the unique factorization theorem. He also gave the first proof of the law of quadratic reciprocity, a deep result previously glimpsed by Euler. To expedite his work, Gauss introduced the idea of congruence among numbers—i.e., he defined a and b to be congruent modulo m (written a ≡ b mod m) if m divides evenly into the differencea − b. For instance, 39 ≡ 4 mod 7. This innovation, when combined with results like Fermat's little theorem, has become an indispensable fixture of number theory.

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