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Study of Number Theory and relationship between different types of numbers and study of types of questions that can be solved

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Introduction : Number theory is the study of the set of positive whole numbers 1, 2, 3, 4, 5, 6, 7, . . . , which are often called the set of natural numbers. We will especially want to study the relationships between different sorts of numbers. Since ancient times, people have separated the natural numbers into a variety of different types . Number theory, also known as higher arithmetic, is a branch of mathematics concerned with the properties of integer s, rational number s, irrational number s, and real number s. Sometimes the discipline is considered to include the imaginary and complex numbers as well.

This is study of the integers, is one of the oldest and richest branches of mathematics. Its basic concepts are those of divisibility, prime numbers, and integer solutions to equations -- all very simple to understand, but immediately giving rise to some of the best known theorems and biggest unsolved problems in mathematics. The Theory of Numbers is also a very interdisciplinary subject. Ideas from combinatorics (the study of counting), algebra, and complex analysis all find their way in, and eventually become essential for understanding parts of number theory. Indeed, the greatest open problem in all mathematics, the Riemann Hypothesis, is deeply tied into Complex Analysis. But never fear, just start right into *Elementary Number Theory*, one of the warmest invitations to pure mathematics, and one of the most surprising areas of applied mathematics

Formally, numbers are represented in terms of set s; there are various schemes for doing this. However, there are other ways to represent numbers -- for example, as angles, as points on a line, as points on a plane, or as points in space.

The integers and rational numbers can be symbolized and completely defined by numeral s. The system of numeration commonly used today was developed from systems used in Arab texts, although some scholars believe they were first used in India.

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Relationship between different types of numbers : The relationships between different sorts of numbers. Since ancient times, people have separated the natural numbers into a variety of different types. Here are some familiar and not-so-familiar examples:

- odd 1, 3, 5, 7, 9, 11, . . .
- even 2, 4, 6, 8, 10, . . .
- square 1, 4, 9, 16, 25, 36, . . .
- cube 1, 8, 27, 64, 125, . . .
- prime 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, . . .
- composite 4, 6, 8, 9, 10, 12, 14, 15, 16, . . .
- 1 (modulo 4) 1, 5, 9, 13, 17, 21, 25, . . .
- 3 (modulo 4) 3, 7, 11, 15, 19, 23, 27, . . .
- triangular 1, 3, 6, 10, 15, 21, . . .
- perfect 6, 28, 496, . . .
- Fibonacci 1, 1, 2, 3, 5, 8, 13, 21, . . .

Many of these types of numbers are undoubtedly already known to you. Others, such as the “modulo 4” numbers, may not be familiar. A number is said to be congruent to 1 (modulo 4) if it leaves a remainder of 1 when divided by 4, and similarly for the 3 (modulo 4) numbers. A number is called triangular if that number of pebbles can be arranged in a triangle, with one pebble at the top, two pebbles in the next row, and so on. The Fibonacci numbers are created by starting with 1 and 1. Then, to get the next number in the list, just add the previous two.
Finally, a number is perfect if the sum of all its divisors, other than itself, adds back up to the 7 original number. Thus, the numbers dividing 6 are 1, 2, and 3, and 1 + 2 + 3 = 6. Similarly, the divisors of 28 are 1, 2, 4, 7, and 14, and 1 + 2 + 4 + 7 + 14 = 28.

Some Typical Number Theoretic Questions :

The main goal of number theory is to discover interesting and unexpected relationships between different sorts of numbers and to prove that these relationships are true. In this section we will describe a few typical number theoretic problems, some of which we will eventually solve are as

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- ***Sums of Squares I***. : Can the sum of two squares be a square? The answer is clearly "YES"; for example $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$. These are examples of Pythagorean triples..
- ***Sums of Higher Powers***. Can the sum of two cubes be a cube? Can the sum of two fourth powers be a fourth power? In general, can the sum of two n th powers be an n^{th} power? The answer is "NO." This famous problem, called Fermat's Last Theorem, was first posed by Pierre de Fermat in the seventeenth century, but was not completely solved until 1994 by Andrew Wiles..
- ***Infinitude of Primes***. A prime number is a number p whose only factors are 1 and p.
    - Are there infinitely many prime numbers?
    - Are there infinitely many primes that are 1 modulo 4 numbers?
    - Are there infinitely many primes that are 3 modulo 4 numbers?

    The answer to all these questions is "YES."
- ***Sums of Squares II***. Which numbers are sums of two squares? It often turns out that questions of this sort are easier to answer first for primes, so we ask which (odd) prime numbers are a sum of two squares. For example

    3 = NO,

    $5 = 1^2 + 2^2$,

    7 = NO,

    11 = NO,

    $13 = 2^2 + 3^2$,

    $17 = 1^2 + 4^2$,

    19 = NO,

    23 = NO,

    $29 = 2^2 + 5^2$,

    31 =NO,

    $37 = 1^2 + 6^2$, . . .

This list leads to the conjecture that p is a sum of two squares if it is congruent to 1 (modulo 4). In other words, p is a sum of two squares if it leaves a remainder of 1 when divided by 4, and it is not a sum of two squares if it leaves a remainder of 3.

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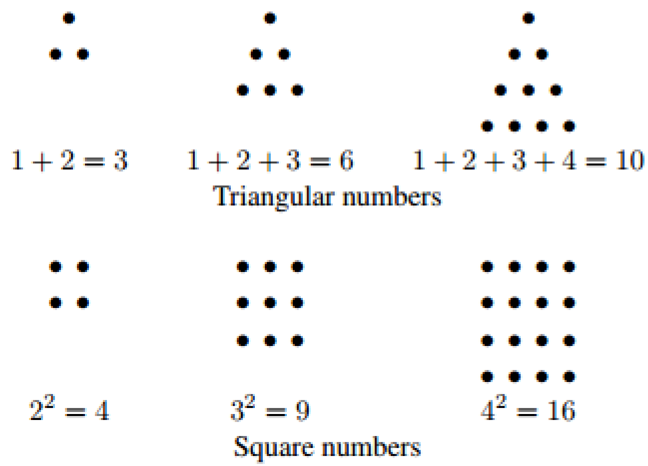
- ***Number Shapes***. The square numbers are the numbers 1, 4, 9, 16, . . . that can be arranged in the shape of a square. The triangular numbers are the numbers 1, 3, 6, 10, . . . that can be arranged in the shape of a triangle. The first few triangular and square numbers are illustrated in Figure



A natural question to ask is whether there are any triangular numbers that are also square numbers (other than 1). The answer is "YES," the smallest example being 36 = 62 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8.

Conclusion : We have now seen some of the types of questions that are studied in the Theory of Numbers. How does one attempt to answer these questions? The answer is that Number Theory is partly experimental and partly theoretical. The experimental part normally comes first; it leads to questions and suggests ways to answer them. The theoretical part follows; in this part one tries to devise an argument that gives a conclusive answer to the questions.

In summary, here are the steps to follow:

1. Accumulate data, usually numerical, but sometimes more abstract in nature.
2. Examine the data and try to find patterns and relationships.
3. Formulate conjectures (i.e., guesses) that explain the patterns and relationships. These are frequently given by formulas.
4. Test your conjectures by collecting additional data and checking whether the new information fits your conjectures.

5. Devise an argument (i.e., a proof) that your conjectures are correct. All five steps are important in number theory and in mathematics. The true test of a scientific theory is its ability to predict the outcome of experiments that have not yet taken place. In other words, a scientific theory only becomes plausible when it has been tested against new data. This is true of all real science. In mathematics one requires the further step of a proof, that is, a logical sequence of assertions, starting from known facts and ending at the desired statement.

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