© INTERNATIONAL JOURNAL FOR RESEARCH PUBLICATION & SEMINAR
ISSN: 2278-6848 | Volume: 07 Issue: 04 | July - September 2016
Paper is available at www.jrps.in | Email : info@jrps.in

Study of Mathematical Modeling, principles and Methods

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Introduction : Mathematical modeling is a principled activity that has both principles behind it and methods that can be successfully applied. The principles are over-arching or meta-principles phrased as questions about the intentions and purposes of mathematical modeling. These meta-principles are almost philosophical in nature. modeling is an activity, a cognitive activity in which we think about and make models to describe how devices or objects of interest behave.

There are many ways in which devices and behaviors can be described. We can use words, drawings or sketches, physical models, computer pro- grams, or mathematical formulas. In other words, the modeling activity can be done in several languages, often simultaneously. Since we are par- ticularly interested in using the language of mathematics to make models,

Mathematical Model Mathematical model is an representation in mathematical terms of the behavior of real devices and objects, Since the modeling of devices and phenomena is essential to both engi- neering and science, engineers and scientists have very practical reasons for doing mathematical modeling. In addition, engineers, scientists, and mathematicians want to experience the sheer joy of formulating and solving mathematical problems.

Mathematical Modeling and the Scientific Method : In an elementary picture of the scientific method (see Figure 1.1), we identify a “real world” and a “conceptual world.” The external world is the one we call real; here we observe various phenomena and behaviors, whether natural in origin or produced by artifacts. The conceptual world is the world of the mind—where we live when we try to understand what is going on in that real, external world. The conceptual world can be viewed as having three stages: observation, modeling, and prediction.

In the observation part of the scientific method we measure what is happening in the real world. Here we gather empirical evidence and “facts on the ground.” Observations may be direct, as when we use our senses, or indirect, in which case some measurements are taken to indicate

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through some other reading that an event has taken place. For example, we often know a chemical reaction has taken place only by measuring the product of that reaction.

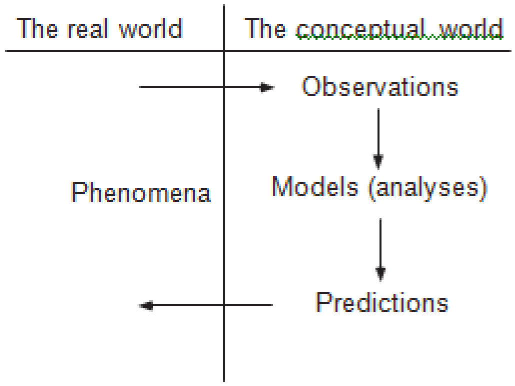

Figure 1.1 An elementary depiction ofthe scientific method that shows how our conceptual models ofthe world are related to observations made within that real world (Dym and Ivey, 1980).

Principles of Mathematical Modeling :

Mathematical modeling is a principled activity that has both principles behind it and methods that can be successfully applied. The principles are over-arching or meta-principles phrased as questions about the intentions and purposes of mathematical modeling. These meta-principles are almost philosophical in nature. We will now outline the principles, and in the next section we will briefly review some of the methods.

A visual portrayal of the basic philosophical approach is shown in Figure 1.2.

These methodological modeling principles are also captured in the following list of questions and answers:

• Why? What are we looking for? Identify the need for the model.
• Find? What do we want to know? List the data we are seeking.
• Given? What do we know? Identify the available relevant data.
• Assume? What can we assume? Identify the circumstances that apply.
• How? How should we look at this model? Identify the governing physical principles.
• Predict? What will our model predict? Identify the equations that will be used, the calculations that will be made, and the answers that will result.
• Valid? Are the predictions valid? Identify tests that can be made to validate the model, i.e., is it consistent with its principles and assumptions?

• Verified? Are the predictions good? Identify tests that can be made to verify the model, i.e., is it useful in terms of the initial reason it was done?

• Improve? Can we improve the model? Identify parameter values that are not adequately known, variables that should have been included, and/or assumptions/restrictions that could be lifted. Implement the iterative loop that we can call "model-validate-verify-improve-predict."

• Use? How will we exercise the model? What will we do with the model?

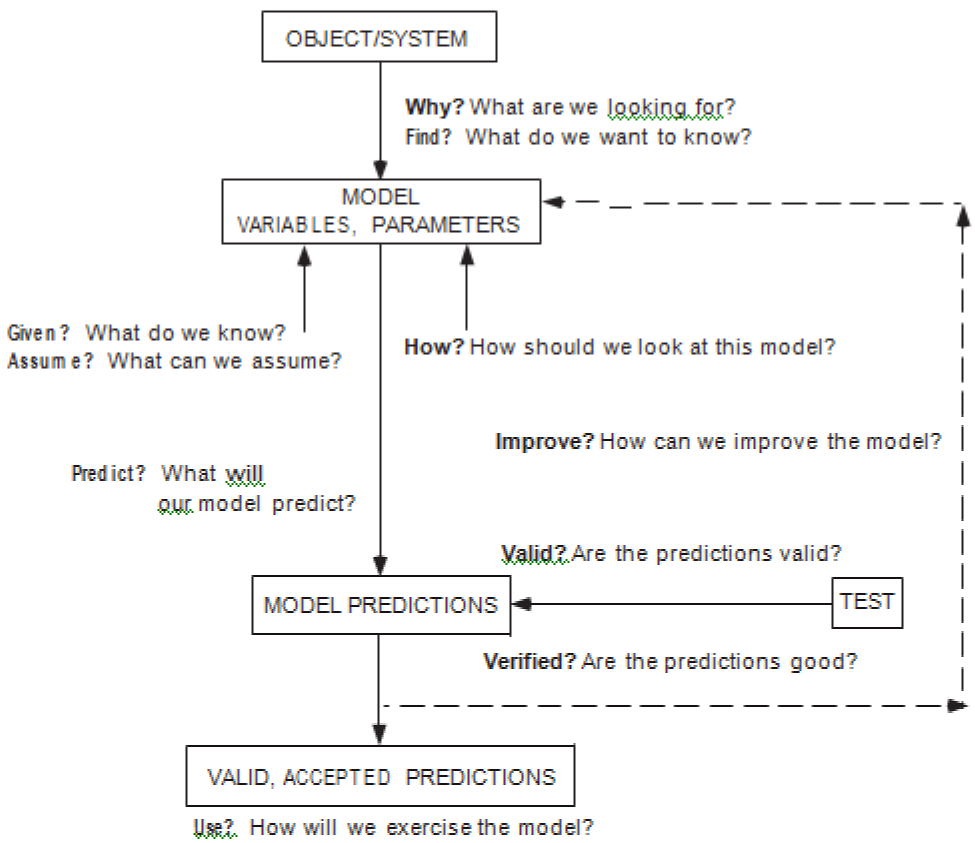

Figure 1.2 A first-order view of mathematical modeling that shows how the questions asked in a principled approach to building a model relate to the development ofthat model (inspired by Carson and Cobelli, 2001).

Having a clear picture of why the model is wanted or needed is of prime importance to the model-building enterprise. Suppose we want to estimate how much power could be generated by a dam on a large river, say a dam located at The Three Gorges on the Yangtze River in Hubei Province in the People's Republic of China. For a first estimate of the available power, we wouldn't need to model the dam's thickness or the strength of its founda- tion. Its height, on the other hand, would be an essential parameter of a power model, as would some model and

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estimates of river flow quantities. If, on the other hand, we want to design the actual dam, we would need a model that incorporates all of the dam's physical characteristics (e.g., dimensions, materials, foundations) and relates them to the dam site and the river flow conditions. Thus, defining the task is the first essential step in model formulation.

If we find that our model is inadequate or that it fails in some way, we then enter an iterative loop in which we cycle back to an earlier stage of the model building and re-examine our assumptions, our known parameter values, the principles chosen, the equations used, the means of calculation, and so on. This iterative process is essential because it is the only way that models can be improved, corrected, and validated.

Some Methods of Mathematical Modeling

Some of the mathematical techniques we can use to help answer the philosophical questions above mentioned. These mathemati- cal principles include: dimensional homogeneity, abstraction and scaling, conservation and balance principles, and consequences of linearity.

- ***Dimensional Homogeneityand Consistency***

There is a basic, yet very powerful idea that is central to mathematical modeling, namely, that every equation we use must be dimensionally homo- geneous or dimensionally consistent. It is quite logical that every term in an energy equation has total dimensions of energy, and that every term in a balance of mass should have the dimensions of mass. This statement provides the basis for a technique called dimensional analysis.

- ***Abstraction and Scaling***

An important decision in modeling is choosing an appropriate level of detail for the problem at hand, and thus knowing what level of detail is prescribed for the attendant model. This process is called abstraction and it typically requires a thoughtful approach to identifying those phenomena on which we want to focus, that is, to answering the fundamental question about why a model is being sought or developed.

For example, a linear elastic spring can be used to model more than just the relation between force and relative extension of a simple coiled spring, as in an old-fashioned butcher's scale or an automobile spring. It can also be used to model the static and dynamic behavior of a tall building, perhaps to model wind loading, perhaps as part of analyzing how the building would respond to an earthquake. In these examples, we can use a very abstract model by subsuming various details within the parameters of that model.

In addition, as we talk about finding the right level of abstraction or the right level of detail, we are simultaneously talking about finding the right scale for the model we are developing. For example, the spring can be used at a much smaller, micro scale to model atomic bonds, in contrast with the macro level for buildings. The notion of scaling includes several ideas, including the effects of geometry on scale, the relationship of function to scale, and the role of size in determining limits—all of which are needed to choose the right scale for a model in relation to the "reality" we want to capture.

- ***Conservation and Balance Principles***

When we develop mathematical models, we often start with statements that indicate that some property of an object or system is being conserved. For example, we could analyze the motion of a body moving on an ideal, frictionless path by noting that its energy is conserved. Sometimes, as when we model the population of an animal colony or the volume of a river flow, we must balance quantities, of individual animals or water volumes, that cross a defined boundary. We will apply balance or conservation principles to assess the effect of maintaining or conserving levels of important physi- cal properties. Conservation and balance equations are related—in fact, conservation laws are special cases of balance laws.

- ***Constructing Linear Models***

Linearity is one of the most important concepts in mathematical model- ing. Models of devices or systems are said to be linear when their basic equations—whether algebraic, differential, or integral—are such that the magnitude of their behavior or response produced is directly proportional to the excitation or input that drives them. We apply linearity when we model the behavior of a device or system that is forced or pushed by a complex set of inputs or excitations. We obtain the response of that device or system to the sum of the individual inputs by adding or superposing the separate responses of the system to each indi- vidual input. This important result is called the principle of superposition.

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