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ISSN: 2278-6848 | Volume: 06 Issue: 07 | October-December 2015

Modern Trends in Mathematical Research

Ravinder, Lecturer in Mathematics, GGSSS, Naguran

In this lecture I will survey some recent trends in mathematics education, by examining the evolution of the study of teaching and learning of algebra. Even though the field of research in mathematics education is small by comparison with the major disciplines in science and humanities it is now too large to make strong claims about trends and findings, so a narrowing is essential. Algebra education is a good choice because it has been a particularly active field.

Mathematics research in India as reflected by paper in deed is quantified and mapped. Statistics, quantum theory and general topology are three sub fields contributing the most to India's output to mathematics research followed by special functions, economics and operations research and relativity theory.

More importantly algebra education is a good choice because many of the major concerns of mathematics education as a whole also impact on algebra education. I see mathematics education as essentially a practical discipline where the underlying goal is always to promote better learning of mathematics by students.

Of course there many subtleties of what mathematics should be learned and why, in whose interest is it that students learn. How achievement is measured etc. but the discipline of mathematics education is under pinned by a faith that a good education in mathematics benefits both the individual and society.

In mathematics especially in ring theory, the simple modulus over a ring R is the left modulus over R that have no non - zero proper sub modulus. Equivalently a module M is simple iff every cyclic sub module generated by a non zero element of M equals M.

Properties of Simple Modulus :

The simple modulus are precisely the modulus of length 1, this is a reformulation of the definition. Every simple modulus is indecomposable but the converse is not true.

Every simple modulus is cyclic i.e. it is generated by one element. Not every module has a simple sub module consider for instance Z-module.

Let M and N be the modulus over the same ring, and let f : M - N be a module homomorphism. If M is simple than f either the zero homomorphism or injective because the kernel of f is sub module of M. If N is simple than f is either zero homomorphism or surjective because the image of f is sub module of N. If M = N then f is endomorphism of M and if M is simple then the prior to statements imply that f is either the zero homomorphism or an isomorphism.

Consequently the endomorphism ring of any simple module is a division ring this result is known as Schur's lemma.

The converse of Schur's lemma is not true in general for example Z module Q is not simple but its endomorphism ring is isomorphic to the field Q.

In mathematics, especially in the field of abstract algebra known as module theory, a semi simple module or completely reducible module is a type of module that can be

© INTERNATIONAL JOURNAL FOR RESEARCH PUBLICATION & SEMINAR
ISSN: 2278-6848 | Volume: 06 Issue: 07 | October-December 2015

understood easily from its parts. A ring that is semi simple module over it self is known as artinian semi simple ring. Some important rings such that group rings of finite groups over fields of characteristic zero are semi simple rings. Artinian ring is initially understood via its largest semi simple quotient.

Semi simple modulus is that modulus that can be written as a sum of simple sub modulus.

A semi simple module M over a ring R can also be thought of as a ring homomorphism from R into the ring of abelian group endomorphism of M. The image of this homomorphism is a semi primitive ring, and every semi primitive ring is isomorphic to such an image.

The endomorphism ring of a semi simple module is not only semi primitive, but also von Neumann regular.

A ring is said to be (left) – semi simple if it is semi simple as a left module over itself. Surprisingly, a left-semi simple ring is also right-semi simple and vice versa. The left/right distinction is therefore unnecessary, and one can speak of semi simple rings without ambiguity.

A semi simple ring may be characterized in terms of homological algebra: namely, a ring R is semi simple if and only if any short exact sequence of left (or right) R-modules splits. In particular, any module over a semi simple ring is injective and projective. Since "projective" implies "flat", a semi simple ring is a von Neumann regular ring.

Semi simple rings are of particular interest to algebraists. For example, if the base ring R is semi simple, then all R-modules would automatically be semi simple. Furthermore, every simple (left) R-module is isomorphic to a minimal left ideal of R, that is, R is a left Kasch ring.

Semi simple rings are both Artinian and Noetherian. From the above properties, a ring is semi simple if and only if it is Artinian and its Jacobson radical is zero.

If an Artinian semi simple ring contains a field as a central subring, it is called a semi simple algebra.

Examples :

A commutative semi simple ring is a finite direct product of fields. A commutative ring is semi simple if and only if it is artinian and reduced.

If k is a field and G is a finite group of order n, then the group ring k[G] is semi simple if and only if the characteristic of k does not divide n. This is Maschke's theorem, an important result in group representation theory.

By the Artin–Wedderburn theorem, a unital Artinian ring R is semi simple if and only if it is (isomorphic to) $M_{n_1}(D_1) \times M_{n_2}(D_2) \times \ldots \times M_{n_r}(D_r)$, where each D_i is a division ring and each n_i is a positive integer, and $M_n(D)$ denotes the ring of n-by-n matrices with entries in D.

Simple rings :

One should beware that despite the terminology, not all simple rings are semi simple. The problem is that the ring may be "too big", that is, not (left/right) Artinian. In fact, if R is a simple ring with a minimal left/right ideal, then R is semi simple.

Classic examples of simple, but not semi simple, rings are the Weyl algebras, such as the Q algebra

A = Q [x, y]/(xy - yx - 1)

Which is a simple noncommutative domain. These and many other nice examples are discussed in more detail in several noncommutative ring theory texts, including chapter 3 of Lam's text, in which they are described as nonartinian simple rings. The module theory for the Weyl algebras is well studied and differs significantly from that of semi simple rings.

Reference :

1. Herstein, Non commutative ring theory lemma 1.1.3
2. Sengupta 2012, p.125