Graph Theory, its characteristics and applications : A study

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Abstract : A graph is a structure amounting to a set of objects in which some pairs of the objects are in some sense "related". The objects of the graph correspond to vertices and the relations between them correspond to edges. A graph is depicted diagrammatically as a set of dots depicting vertices connected by lines or curves depicting edges. Formally, "A graph $G = (V,E)$ consists of V, a non-empty set of vertices (or nodes) and E, a set of edges. Each edge has either one or two vertices associated with it, called its endpoints."

**Key Words :** Graph, Node, Vertex, Edges

Introduction :

A graph is a data structure that is defined by two components : A node or a vertex.

An edge E or ordered pair is a connection between two nodes u,v that is identified by unique pair(u,v). The pair (u,v) is ordered because (u,v) is not same as (v,u) in case of directed graph.The edge may have a weight or is set to one in case of unweighted graph.

Consider the given below graph,
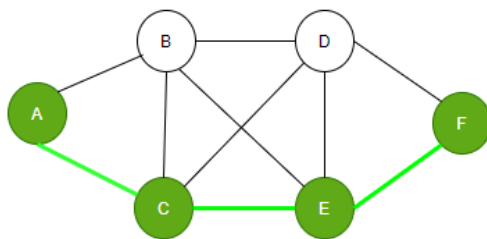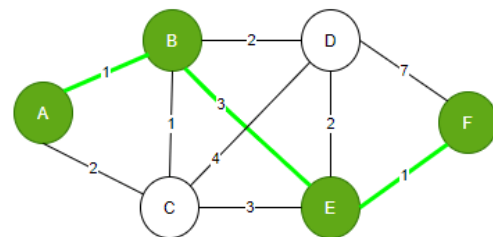
UnWeighted Graph   Weighted Graph

Applications:

Graph is a data structure which is used extensively in our real-life.

- *Social Network*: Each user is represented as a node and all their activities, suggestion and friend list are represented as an edge between the nodes.

- *Google Maps:* Various locations are represented as vertices or nodes and the roads are represented as edges and graph theory is used to find shortest path between two nodes.

- *Recommendations on e-commerce websites:* The "Recommendations for you" section on various e-commerce websites uses graph theory to recommend items of similar type to user's choice.

- Graph theory is also used to study molecules in chemistry and physics.

Properties of Graph

- The starting point of the network is known as root.

- When the same types of nodes are connected to one another, then the graph is known as an assortative graph, else it is called a disassortative graph.

- A cycle graph is said to be a graph that has a single cycle.

- When all the pairs of nodes are connected by a single edge it forms a complete graph.

- A graph is said to be in symmetry when each pair of vertices or nodes are connected in the same direction or in the reverse direction.

- When a graph has a single graph, it is a path graph.

Trees, Degree and Cycle of Graph

There are certain terms that are used in graph representation such as Degree, Trees, Cycle, etc.

Trees: A tree in a graph is the connection between undirected networks which are having only one path between any two vertices. It was introduced by British mathematician Arthur Cayley in 1857. The graph trees have only straight lines between the nodes in any specific direction but do not have any cycles or loops. Therefore trees are the directed graph.

Degree: A degree in a graph is mentioned to be the number of edges connected to a vertex. It is denoted $deg(v)$, where v is a vertex of the graph. So basically it the measure of the vertex.

Cycle: A cycle is a closed path in a graph that forms a loop. When the starting and ending point is the same in a graph that contains a set of vertices, then the cycle of the graph is formed. When there is no repetition of the vertex in a closed circuit, then the cycle is a simple cycle. The cycle graph is denoted by C_n.

- A cycle that has an even number of edges or vertices is called Even Cycle.

- A cycle that has an odd number of edges or vertices is called Odd Cycle.

Types of graphs:

Directed graph:
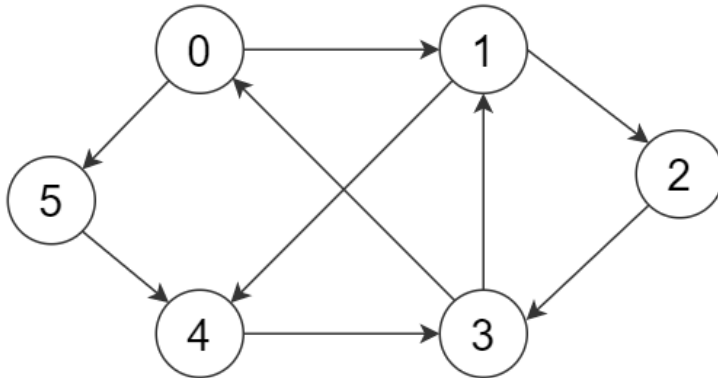
A graph in which the direction of the edge is defined to a particular node is a directed graph.

Directed Acyclic graph:

In graph theory, a graph refers to a set of vertices which are connected by lines called edges. In a directed graph or a digraph, each edge is associated with a direction from a start vertex to an end vertex. If we traverse along the direction of the edges and we find that no closed loops are formed along any path, we say that there are no directed cycles. The graph formed is a directed acyclic graph.

A DAG is always topologically ordered, i.e. for each edge in the graph, the start vertex of the edge occurs earlier in the sequence than the ending vertex of the edge.

Application : Critical game analysis, expression tree evaluation, game evaluation.

Tree: A tree is just a restricted form of graph. That is, it is a DAG with a restriction that a child can have only one parent.

Undirected graph:
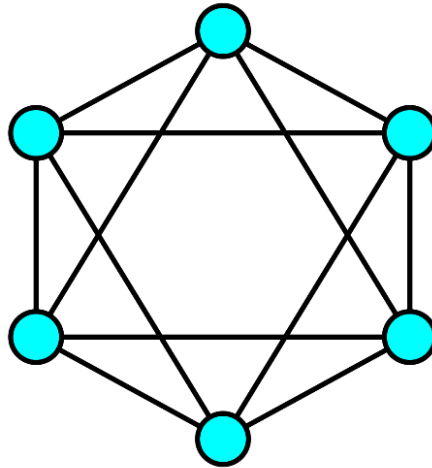
In an undirected graph the edges are bidirectional, with no direction associated with them. Hence, the graph can be traversed in either direction. The absence of an arrow tells us that the graph is undirected.

Connected graph: A graph is connected when there is a path between every pair of vertices. In a connected graph there is no unreachable node.

Complete graph: A graph in which each pair of graph vertices is connected by an edge. In other words, every node 'u' is adjacent to every other node 'v' in graph 'G'. A complete graph would have n(n-1)/2 edges.

Biconnected graph: A connected graph which cannot be broken down into any further pieces by deletion of any vertex. It is a graph with no articulation point.

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